

Quiz 4
20 minutes

Question 1

State **True of False**. Justification is not required. In the problems the bold font for example \mathbf{F} indicates a vector, whereas the normal font for example f indicates a scalar. In most problems D is a region that is not necessarily simply connected, unless explicitly stated.

1. If \mathbf{F} is conservative on D and $\mathbf{F} = P\mathbf{i} + Q\mathbf{j}$ (where P and Q are continuously differentiable) then $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D .
2. If P and Q are continuously differentiable and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D then \mathbf{F} is conservative on D .
3. If C is a closed curve and f is a function, then $\int_C f ds = 0$.
4. If there exists a closed curve C inside a region D such that $\int_C \mathbf{F} \cdot d\mathbf{r} = 0$ then \mathbf{F} is conservative on D .
5. A region is simply connected if any two points in D can be joined by a curve that stays inside D .
6. If $\int_C f ds = 0$ for any function f , then C is a closed curve.
7. If $\nabla \times \mathbf{F} = 0$, then \mathbf{F} is conservative.
8. If \mathbf{F} is source-free (for example a magnetic field) then $\nabla \cdot \mathbf{F} = 0$.
9. If \mathbf{F} is a vector field then $\nabla \cdot \mathbf{F}$ is a scalar field.

10. If \mathbf{F} is a vector field then $\nabla \times \mathbf{F}$ is a vector field.
11. If $\nabla \times \mathbf{F} \neq 0$, then \mathbf{F} is not conservative.
12. If \mathbf{F} is conservative, then $\nabla \cdot \mathbf{F} = 0$.
13. The work done by a conservative force field in moving a particle around a closed path is zero.
14. If \mathbf{F} and \mathbf{G} are two vector fields and $\nabla \times \mathbf{F} = \nabla \times \mathbf{G}$, then $\mathbf{F} = \mathbf{G}$.
15. If f has continuous partial derivatives on \mathbb{R}^3 and C is any circle then $\int_C \nabla f \cdot d\mathbf{r} = 0$.
16. If \mathbf{F} is a vector field on \mathbb{R}^3 then $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.
17. $\nabla \times (\nabla f)$ is always non zero.
18. $\text{curl}(\text{div}(\mathbf{F}))$ is not a meaningful expression.
19. If \mathbf{F} is conservative on a region D then there is some function f on D such that $\nabla f = \mathbf{F}$.
20. If work done by a force \mathbf{F} on an object moving along a curve is W , then if the object moves along the curve in the opposite direction the work done by \mathbf{F} will be $-W$.

SCRATCH PAPER